INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS, VOL. 1, 3–16 (1981)

A VECTOR UPSTREAM DIFFERENCING SCHEME FOR PROBLEMS IN FLUID FLOW INVOLVING SIGNIFICANT SOURCE TERMS IN STEADY-STATE LINEAR SYSTEMS

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SUMMARY

This paper considers a finite difference scheme for modelling the convection/diffusion equation in strongly convective flow regimes including circumstances in which significant source terms are present.

The main objective is to provide an alternative approach to central and/or upwind difference methods which for various reasons are unsatisfactory. To illustrate the main features of the scheme, an assessment of its accuracy is made by means of a Taylor expansion analysis and a study of its performance in two model problems. As a demonstration of its generality for use in large-scale practical problems, some numerical results are presented for the prediction of the temperature distribution in a flow through a partially blocked heated rod bundle.

The main conclusions are that in almost all practical circumstances results obtained using the scheme are not susceptible to false diffusion or spatial oscillations, which are, respectively, the inherent weaknesses in many upwind and central difference scheme formulations, and in general its use results in improved overall accuracy.

KEY WORDS Vector Upstream Differencing Scheme Significant Sources False Diffusion Spatial Oscillation Steady-state Linear Systems Convective Flow Rod Bundle Geometry Temperature Prediction

INTRODUCTION

In a wide range of physical applications it is necessary to solve transport equations describing the convection and diffusion of various fluid properties (e.g. momentum, heat, chemical concentration, etc.). Frequently the flows are complex and multi-dimensional and therefore finite difference or finite element techniques provide the only methods of solution.

In finite difference calculations the most commonly used methods employ either central difference schemes (CDS)[1,2] or upwind difference methods (UDS)[1,2] depending on the range of Reynolds or Peclet numbers of interest. In many practical problems the Reynolds (Peclet) numbers are high and it is well known that in these circumstances central difference methods give rise to non-physical spatial oscillations.[3,4] In addition, they are unsuitable for iterative solution methods.[1]

To overcome this problem the upwind difference scheme is frequently invoked which produces oscillation-free solutions[3] and is well suited to iterative solution methods.[1] However, it does have major shortcomings. Firstly, if significant gradients of the dependent variable exist normal to the streamline, and if the flow is oblique to the mesh, then considerable errors will arise as a consequence of false diffusion.[5] Secondly, if source terms are present then errors can arise as a consequence of the upstream approximation.[6] Attempts have been made to improve accuracy by the use of a hybrid differencing scheme HDS[1,2] which reduces to CDS or UDS depending on whether the local mesh Reynolds (Peclet) number is less than

0271–2091/81/010003–14$01.00

Received 19 March 1980

or greater than 2, respectively. Clearly, however, by definition of the HDS the problems of false diffusion and mesh displacement of the solution will still exist at mesh Reynolds (Peclet) numbers larger than 2.

This paper presents both an analysis and numerical applications of a vector upstream differencing scheme (VUDS) which was produced to eliminate these shortcomings. This scheme employs an upstream approximation along a direction dictated by the local streamline, and in this respect has similarities with a scheme produced by Raithby.[6] However, in addition it contains a source correction term which aims to eliminate mesh displacement effects inherent in upstream weighted difference schemes. Reference should also be made here to the work of Leohard,[16] where pure convection with source term is successfully treated and in the finite element context, a Streamline Upwind Procedure which avoids cross-wind diffusion has been proposed by Hughes *et al.*[16] The analysis as now described will apply only to the steady state, although generalisation of the treatment to include transient problems is currently in progress.

CONSERVATION EQUATION

The conservation equation for the transport of the variable ϕ by convection and diffusion may be written in the following form:

$$\nabla\,.\,(\rho\mathbf{u}\phi-\Gamma\nabla\phi)=S \tag{1}$$

where $\rho \equiv$ density, $\mathbf{u}=(u, v, w)$ velocity, Γ is the exchange coefficient and S is the source. In control volume analysis, equation (1) is integrated over a control volume to give

$$\int_S (\rho\mathbf{u}\phi-\Gamma\nabla\phi)\,.\,\mathbf{n}\,\mathrm{d}S=\int S\,\mathrm{d}V \tag{2}$$

where $\mathbf{n}$ denotes an outward normal, and the finite difference approximations are applied to the surface face fluxes. In general, the diffusion terms provide a stabilising influence and the well-known central difference approximation can be applied. In this paper the main point of interest is to describe the VUDS approximation of the convection terms in the high Peclet number regime, and this is given in the next section. Throughout we shall refer to the dimensionless group $\rho uL/\Gamma$, relating convection to diffusion as the Peclet number, implying ϕ represents energy, although the analysis remains general for arbitrary ϕ satisfying equation (1).

VECTOR UPSTREAM DIFFERENCING SCHEME

In describing the VUDS, the transport of ϕ across the $x+$ face only will be considered. Analogous expressions exist for the remaining three faces. Now it is usually argued that if convection dominates then the transport of ϕ by convection depends on upstream conditions and this therefore should be reflected in the locally assumed profile in deriving the scheme.

For example, if there are no sources then the gradient of ϕ in the streamwise direction should be small and the normal gradient will depend on the upstream conditions. This was recognised by Raithby,[6] who derived a scheme based on a locally assumed profile of the form

$$\phi=C_1+C_2 n \tag{3}$$

relative to a local origin at $x+$ (see Figure 1). However, if significant sources are present then the gradient of ϕ along the streamline is not necessarily small. To accommodate this

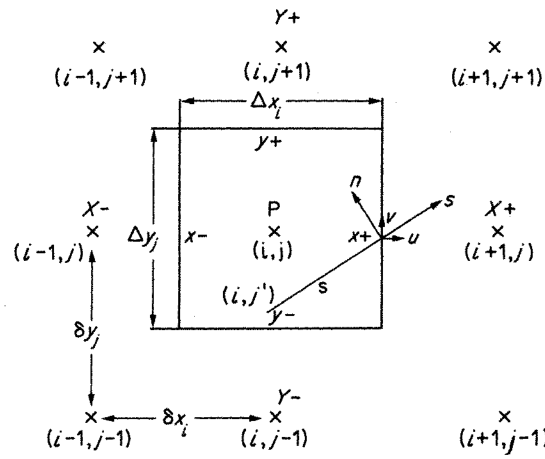

Figure 1. Two-dimensional control volume

phenomenon, the VUDS is derived on the assumption that

$$\phi = C_1 + C_2 n + C_3 s \tag{4}$$

To ensure a linear distribution normal to the streamline this profile is fitted to $\phi_{i,j}$, $\phi_{i,j-1}$. In addition, ignoring curvature effects, we may write (see Figure 1)

$$\phi_{i+\frac{1}{2},j} = \phi_{i,j'} + \oint_{i,j'}^{i+\frac{1}{2},j} (S/\rho u_s)\, \mathrm{d}s \tag{5}$$

where S denotes a source correction to allow for the heat gained by the fluid along the streamline and u_s denotes the stream velocity. The basic assumption behind the VUDS is to consider S as a control volume averaged property, and we take

$$\int_{i,j'}^{i+\frac{1}{2},j} (S/\rho u_s) = (s_1 S_{i,j})/(\rho u_s)_{i+\frac{1}{2},j} \tag{6}$$

By substituting this approximation into equation (5) a third condition is determined and hence the constants C_1, C_2, C_3 can be evaluated. It should be mentioned that this approach contains an approximation. Strictly, the right-hand side should also contain a term $\nabla \cdot \Gamma \nabla \phi$ and an approximation for this term may also be included if desired. Except in very specialised circumstances, such a correction is not necessary and the simplified version has been taken in most practical applications of the VUDS to date.

Physically this approximation of ϕ_{x+} is a definition of ϕ at (i, j') based on linear interpolation between $\phi_{i,j}$ and $\phi_{i,j-1}$, together with a correction term to allow for the upstream approximation. If u, v denote components of u_s in the x, y directions, respectively, and λ_u, λ_v have magnitude unity with the sign of u, v respectively, then for a general flow direction a mathematically compact expression for the convective flux of ϕ across the $x+$ face can be written in the following form:

$$C_{x+} = 2L_{x+}\{(1+J_{x+})((1-K_{x+})\phi_{1+,j} + K_{x+}\phi_{1+,m+}) + (1-J_{x+})\phi_{h+,m+}\} + (s_1 S_{1+,j})(u/u_s)_{i+\frac{1}{2},j}\Delta y_j \tag{7}$$

where

$$h+ = i + \tfrac{1}{2}(1+\lambda_u) \tag{7a}$$

$$k+ = j + \tfrac{1}{2}(1-\lambda_v) \tag{7b}$$

$$1+ = i + \tfrac{1}{2}(1-\lambda_u) \tag{7c}$$

$$m+ = j - \lambda_v \tag{7d}$$

$$f_{k+} = \delta x_{i+1}\,|v|/(2\delta y_{k+}\,|u|) \tag{7e}$$

$$L_{x+} = \tfrac{1}{2}(\rho u \Delta y)_{i+\frac{1}{2},j} \tag{7f}$$

$$J_{x+} = \min(1, f_{k+}^{-1}) \tag{7g}$$

$$K_{x+} = \min(1, f_{k+}) \tag{7h}$$

Equation (7) represents a generalisation of the formula given by Raithby.[6] It reduces to his simpler scheme in the cases

$$J_{x+} = 1 \tag{8}$$

$$S = 0 \tag{9}$$

The first condition is equivalent to restricting the upstream interpolation to the vertical (horizontal) lines joining nodes only, in prescribing the flux of ϕ in the x, (y) directions, taking the end-point nodes if extrapolation is required. This restriction is adequate in many cases but produces some loss in accuracy in circumstances where there is a significant source and where the ϕ distribution is governed by diffusion normal to the streamline—see discussion on the vortex model problem later. However, for other numerical reasons the restriction has been found desirable in certain circumstances. Both these points will be discussed in more detail later.

The flux of ϕ by diffusion across the $x+$ face is taken to be the standard central difference expression

$$D_{x+} = -\Lambda_{x+}(\phi_{X+} - \phi_P) \tag{10}$$

where

$$\Lambda_{x+} = \Gamma_{x+}\Delta y_j/\delta x_{i+1} \tag{10a}$$

The total transport of ϕ across the $x+$ face by convection and diffusion is therefore

$$F_{x+} = C_{x+} + D_{x+} \tag{11}$$

and in two dimensions the VUDS approximation to equation (2) can be written in the form

$$F_{x+} - F_{x-} + F_{y+} - F_{y-} = S_{ij}\,\delta V \tag{12}$$

SOLUTION METHODS

In this section, before discussing questions of accuracy we consider methods available for the solution of VUDS.

The typical equation corresponding to a node P may be written in the form

$$a_P\phi_P = \Sigma(a_{x+}\phi_{X+} + b_{x+}\phi_{1+,m+} + c_{x+}\phi_{h+,m+}) + S_{i,j}\,\delta V \tag{13}$$

where the summation extends over $x+$, $x-$, $y+$, $y-$,

$$a_P = \Sigma(a_{x+} + b_{x+} + c_{x+}) \tag{13a}$$

$$a_{x+} = \Lambda_{x+} + (L_{x+}\sigma/2)(1+J_{x+})(1-K_{x+})(1+\sigma\lambda_u) \tag{13b}$$

$$b_{x+} = L_{x+}\sigma(1+J_{x+})K_{x+} \tag{13c}$$

$$c_{x+} = L_{x+}\sigma(1-J_{x+}) \tag{13d}$$

and

$$\sigma = 1 \text{ for } x- \text{ and } y-; \qquad \sigma = -1 \text{ for } x+ \text{ and } y+.$$

The first point to note is that since the VUDS without source correction reduces to the UDS for flow aligned with the mesh, there is diagonal dominance and solutions themselves are oscillation free, at least in this case. However, the VUDS is not unconditionally diagonally dominant, as can be seen in the case of an equal mesh when the flow is at 45° to the mesh, in which case

$$|a_P| = \tfrac{1}{3} \sum (|a_{x+}| + |b_{x+}| + |c_{x+}|) \tag{14}$$

This means that in order to ensure convergence it is advantageous to rewrite equation (13) into the deferred corrector form

$$a'_P\phi_P - \sum a'_{x+}\phi_{x+} = (a'_P - a_P)\phi_P + \sum \{(a_{x+} - a'_{x+})\phi_{X+} + b_{x+}\phi_{1+,m+} + c_{x+}\phi_{h+,m+}\} + S_P\,\delta V \tag{15}$$

where the dashes denote coefficients obtained by the UDS formulation.

Practical experience has shown that equation (15) may be solved by employing standard techniques to the left-hand side for solving the UDS and treating the right-hand side as a modified source evaluated using values of ϕ obtained from a previous iteration. This method is used in the numerical application considered later in the paper and for linear equations in ϕ has been found satisfactory in all cases.

For nonlinear systems, the convergence is quite acceptable for the simplified version of the VUDS with $J_{x+} = 1$, etc. For the general version convergence, using this method can be rather slow for nonlinear equations and there may be benefits in seeking an alternative method of solution.

ACCURACY

Taylor expansion methods

The nature of the VUDS can to a large extent be predicted by an examination of the leading error terms in Taylor series expansions. In particular we consider the error in the finite difference scheme defined by

$$\varepsilon_{\mathrm{FDS}} = \frac{1}{\delta V}\left\{\mathrm{FD}\left(\int \rho\mathbf{u}\phi \,.\, \mathbf{n}\,\mathrm{d}S\right) - \int \rho\mathbf{u}\phi \,.\, \mathbf{n}\,\mathrm{d}S\right\} \tag{16}$$

where $\mathrm{FD}(\int \rho\mathbf{u}\phi \,.\, \mathbf{n}S)$ denotes the finite difference approximation to $\int \rho\mathbf{u}\phi \,.\, \mathbf{n}\,\mathrm{d}S$.

Now corresponding to the x direction

$$\varepsilon^x_{\mathrm{UDS}} = \tfrac{1}{2}(\lambda_{i-\frac{1}{2}} - \lambda_{i+\frac{1}{2}})\left(\rho u \frac{\partial\phi}{\partial x}\right)_i - \frac{h}{4}(\lambda_{i-\frac{1}{2}} + \lambda_{i+\frac{1}{2}})\frac{\partial}{\partial x}\left(\rho u \frac{\partial\phi}{\partial x}\right) + 0(h^2) \tag{17}$$

for an equal mesh with interval of length h in the notation of Figure 1 (j suffices are dropped).

There is a zero-order error term which only vanishes if the flow components entering and leaving the control volume have the same sign together with the well-known first-order false diffusion error term. Now corresponding to the x direction and with no source correction, $\varepsilon^x_{\mathrm{UDS}}$ is given by

$$\varepsilon^x_{\mathrm{VUDS}} = \overset{(1)}{\tfrac{1}{2}(\lambda_{i-\frac{1}{2}} - \lambda_{i+\frac{1}{2}})\left(\rho u_s \frac{\partial \phi}{\partial s}\right)_i} - \overset{(2)}{\frac{h}{4}(\lambda_{i-\frac{1}{2}} + \lambda_{i+\frac{1}{2}})\frac{\partial}{\partial x}\left(\rho u_s \frac{\partial \phi}{\partial s}\right)_i} \tag{18}$$

which is analogous to $\varepsilon^x_{\mathrm{UDS}}$ but in this case $\rho u\, \partial\phi/\partial x$ is replaced by $\rho u_s\, \partial\phi/\partial s$. This means that the low-order error terms at least do not involve significant terms involving gradients normal to the stream and hence the principle cause of false diffusion is eliminated. If, in addition, the source correction term is added this zero-order error term is eliminated from equation (18). This means, for example, that distributions of ϕ across regions of flow reversal will be more smoothly predicted than with the uncorrected scheme and overall accuracy will be improved.

We next consider the accuracy of VUDS when significant sources are present, by reference to two model problems—a unidirectional flow problem where the profile is dominated by the convection terms and source, and a diffusion problem where the profile is governed by diffusion normal to the streamline.

Unidirectional flow problem

It was shown[6] that the simplified form of the VUDS preserved the profile of the transport of a step change of ϕ for a uniform velocity field which is source free. A similar conclusion applies therefore for the general form of the VUDS for flows satisfying $\frac{1}{2} \leqslant \left|\frac{v}{u}\right| \leqslant 2$ (in, for example, the equal mesh case) in the notation of Figure 1. An independent study indicated that both the general and simplified forms of the VUDS preserved an imposed sinusoidal profile satisfactorily, for a range of flow to mesh inclinations. (The results are not presented here). However in both cases, if source terms are present then the error arising as a consequence of the upstream approximation can mean that the predicted field is displaced. To see this we may consider, for example, a one-dimensional constant mass flow problem with a step source, shown in Figure 2. A staggered mesh is assumed with the ϕ nodes at $x = 0, 1, 2$ and the control volume boundaries at $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$.

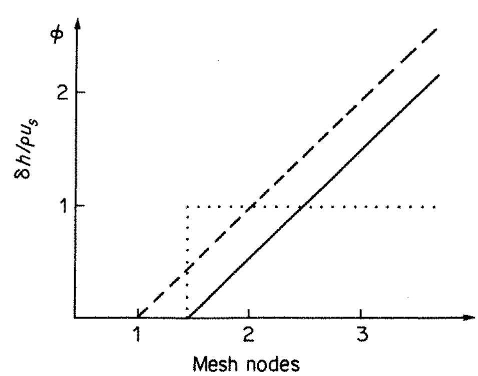

Figure 2. Unidirectional flow with source discontinuity (···· source profile; —— UDS, VUDS without source correction; —— correct solution, VUDS with source correction)

Now with the source correction term, the VUDS gives $\phi_2 = \frac{1}{2}$, the correct solution, although with no correction it would reduce to the UDS and hence predict $\phi_2 = 1$.

This apparently trivial problem serves to illustrate how the convective transport and source terms can become inconsistent if no source correction is made. Such discrepancies can be very important in some circumstances. For example, in a calculation to describe boiling where ϕ represents the energy it is often very important to correctly predict the position of the boiling boundary in determining the flow field.

Vortex model problem

As a second example, results are presented for the predicted ϕ distribution in a vortex consisting of a fluid rotating with constant angular velocity and containing a uniformly distributed source. The ϕ distribution on the boundary was specified and an analytic solution derived. The finite difference solutions were obtained for a 9×9 equal mesh grid in an inscribed square whose centre coincided with the centre of the vortex. The finite difference equations were closed by specifying the values of ϕ on the edges of the square according to the analytic solution. For constant coefficients, equation (1) may be transformed into the dimensionless form

$$\mathrm{Pe}\frac{\partial \phi}{\partial \theta} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) - \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 4 \qquad 0 \leq r \leq 1, \quad 0 \leq \theta \leq 2\pi \tag{19}$$

where the Peclet number is based on the radius and peripheral velocity of the vortex. The general solution[8,9] of equation (1), corresponding to a boundary condition of the form

$$\phi(1, \theta) = \cos \theta \tag{20}$$

is given by the real part of

$$\phi(r, \theta) = \overset{(1)}{1 - r^2} + \overset{(2)}{\bar{J}_1(r\sqrt{(-i\,\mathrm{Pe})})/\bar{J}_1(\sqrt{(-i\,\mathrm{Pe})})e^{i\theta}} \tag{21}$$

where $\bar{J}$ denotes the Bessel function of the first kind.[9]

Term (2) represents the influence of the sinusoidal boundary condition and is superimposed on the familiar parabolic solution, term (1). The problem provides a very severe test for finite difference solutions. The parabolic term produces very steep gradients of ϕ normal to the streamline, and hence any tendency of an upwind scheme to produce false diffusion effects is apparent at even modest Peclet numbers. The influence of term (2) produces a streamwise variation at modest Peclet numbers, thus providing an additional test for the finite difference calculation.

Figure 3 gives analytical ϕ profiles along the diameter $\theta = \pi/4$ for a mesh Peclet number of 40, together with predictions of the full and simplified versions ($J_{x+} = J_{x-} = J_{y+} = J_{y-} = 1$ cf. equation 8) of the VUDS and also the HDS. Figure 3 also shows corresponding errors ε of the three schemes at the vortex centre for a range of mesh Peclet numbers up to 50. Here ε is defined as the difference between the analytic value ϕ_{AS} and the finite difference predictions ϕ_{FDS}.

It is clear that the HDS is dominated by false diffusion at high Peclet numbers and is grossly inadequate. The simplified version of the VUDS, although much more accurate than HDS, still exhibits some false diffusion effects. The more complicated version retains good accuracy throughout the Peclet number range.

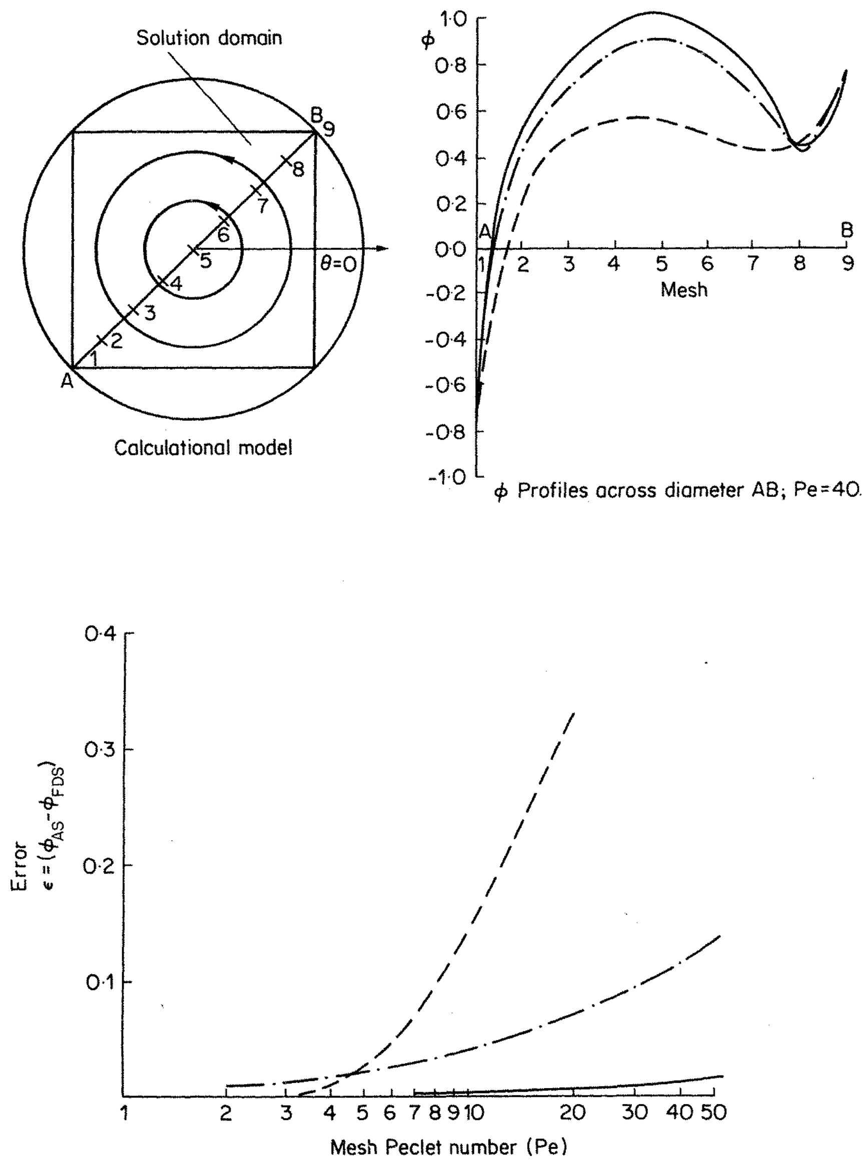

Figure 3. Vortex model problem (— — HDS; ·—· VUDS, simplified version; —— VUDS)

PRACTICAL APPLICATIONS

Although it is necessary and also useful to assess the accuracy of a numerical method by means of theoretical analysis and performance on simple model problems, it is equally important to evaluate its performance on large-scale industrial applications. To meet this latter requirement some results will now be presented for the prediction of temperature in disturbed flows in subchannel geometry which were produced using an application of the VUDS for the prediction of the temperature field in the SABRE code.[10]

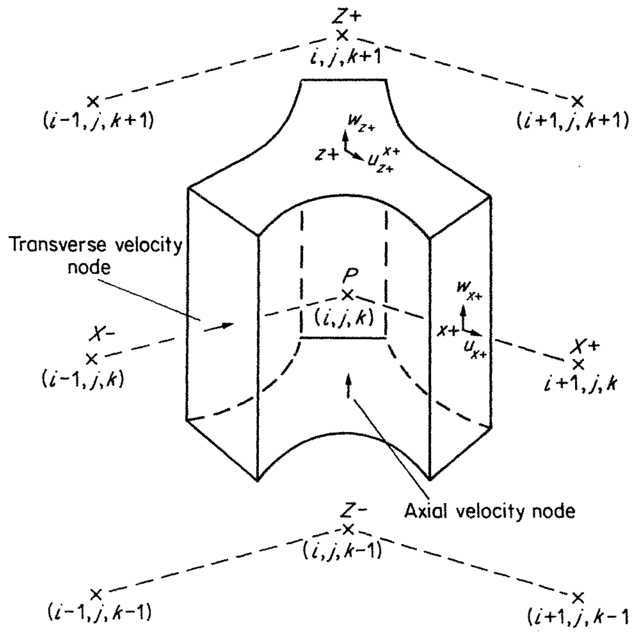

Figure 4. Three-dimensional control volume in subchannel geometry

This code was written to solve the Navier–Stokes and energy equations in subchannel geometry in order to predict events on the formation of a hypothetical flow blockage, and thus to provide information required for fast reactor safety analysis. Such blockages are large enough to produce a recirculation region embedded in an otherwise unidirectional flow. It is therefore necessary that any finite difference representation is accurate in both flow regimes. This is especially important in subchannel codes such as SABRE, where the lateral mesh is determined by the pin pitch and is therefore rather coarse. A final requirement is that the scheme be amenable to iterative solution techniques which are necessary due to the large number of equations which need to be solved, a consequence of the three-dimensional capability of the code.

Before the test problem is discussed, however, we shall describe in brief terms the generalisation of the VUDS to subchannel geometry.

VUDS in subchannel geometry

Attention is focused on a typical control volume positioned between fuel pins whose centres form an equilateral triangular lattice. The basic velocity nodes (u, v, w) are located on the faces of the control volume and are midway between adjacent temperature nodes. The fundamental assumption made in generalising the VUDS is that the flow between any pair of pins is locally two dimensional, thus enabling the treatment of the earlier section to be applied.

To calculate the transverse heat flux, an axial component of velocity w_{x+} is obtained at the location of $x+$ by interpolation from neighbouring axial velocity nodes—see Figure 4. Since a u-node exists at $x+$, F_{x+} can then be calculated according to the two-dimensional treatment given in equation (7). To describe the axial flux F_{z+} is more complicated. Full details are given in Lillington,[11] but the general idea is to define three transverse components of velocity u_{z+}^{x+}, u_{z+}^{x-}, v_{z+}^{y+} (v_{z+}^{y-}, depending on the orientation of the subchannels) at the location of $z+$.

Then again, since a w-node exists at $z+$, u_{z+}^{x+}, and w_{z+} (and similarly the other pairs) define a stream direction and enable a heat flux F_{z+}^{x+} across the axial face to be calculated as in the two-dimensional treatment on the assumption that all the flow is in that direction. F_{z+}^{x-} and F_{z+}^{y+} (F_{z+}^{y-}) are defined similarly. The heat flux F_{z+} is then taken as a weighted sum of these three contributions

$$F_{z+} = \sum \gamma_{x+} F_{z+}^{x+} \tag{22}$$

where

$$\gamma_{x+} = u_{z+}^{x+} / \sum u_{z+}^{x+} \tag{22a}$$

and the summation are over all directions.

An overall heat balance gives an equation similar to equation (13). The solution procedure for the hydrodynamics equations in the SABRE code is based on the SIMPLE procedure.[12] Within this framework the system of equations for the energy are solved by applying an ADI[13] inversion algorithm plane by plane, followed by axial block adjustment[12] to the left-hand side during which the right-hand side remains fixed and determined by values from the previous iteration.

NUMERICAL RESULTS

The importance of eliminating false diffusion will be demonstrated in a partial flow blockage calculation relating to the test section shown in Figure 5. This experiment, on a 281-pin water-cooled bundle, with the central 54 subchannels blocked, was carried out at the Berkeley Nuclear Laboratories of the CEGB.[14] A central region of 91 electrically heated pins rated at 17·5 W/cm^2 was surrounded by an annular region of 192 unheated pins. The pitch is 8·3 mm, the same for heated and unheated pins, but the diameters are 6·6 mm and 6·1 mm, respectively. The blockage was 6 mm thick and covered 7·3% per cent of the total flow area.

The calculation was performed over a 30° sector of the cross-section, taking advantage of the symmetry—see Figure 5. The boundary conditions were specified as horizontally uniform pressure at inlet and outlet of the calculation domain and corresponded to an inlet mean velocity of 6·5 m/sec. The inlet temperature was 20°C.

In assessing the accuracy of any finite difference representation, it is necessary to unscramble which effects are due to weaknesses in the finite difference representation and which effects are due to weaknesses in other aspects of the modelling, notably in geometrical or turbulence model representations. In these results, therefore, the flow equations have been decoupled from the energy calculations and thus any variations are due solely to changes in the heat transport (a detailed description of the hydrodynamics is given in Reference 10). A plot of flow vectors in an axial plane through the subchannels numbers 1 to 8 is shown in Figure 6.

Radial distributions of temperature are shown in Figure 6 for two axial planes, distant d mm downstream of the blockage. The results are plotted in two radial directions—the corner direction from the axis of the test section towards the corner of the hexagonal blockage, and the centre direction from the axis to the centre of one face of the hexagon.

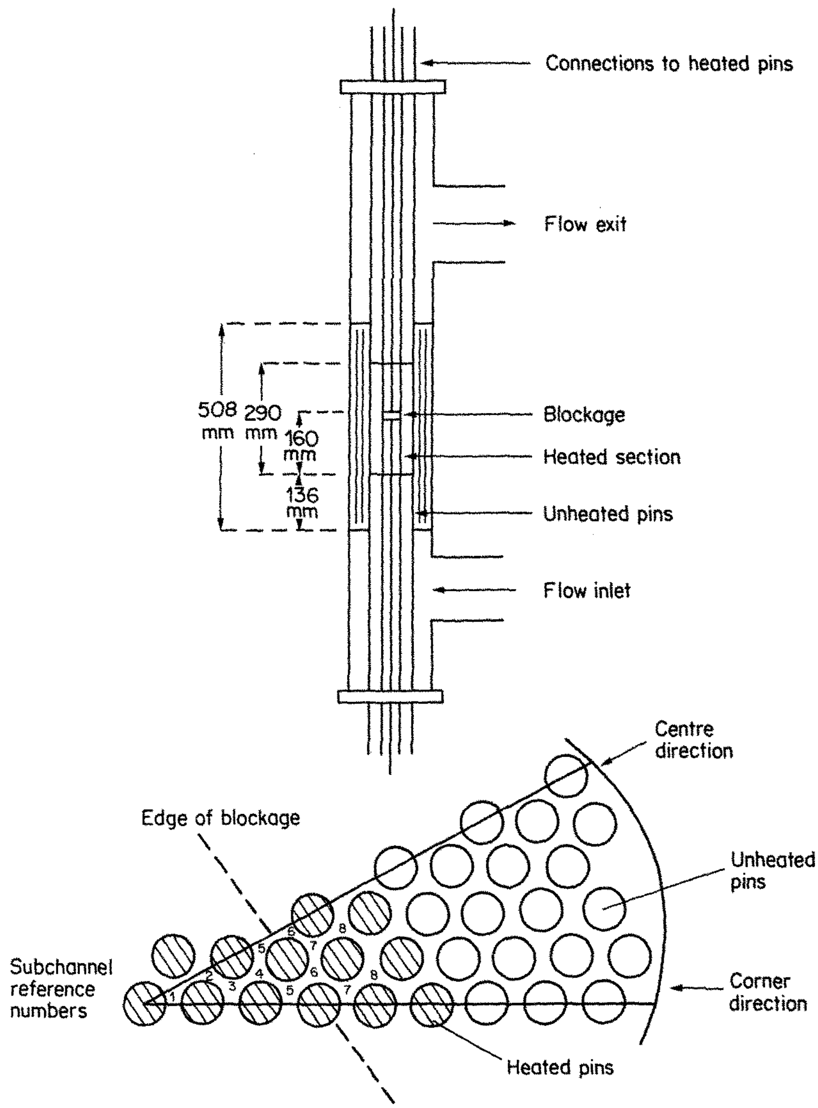

Figure 5. Geometrical details of the test section

The directions are shown in Figure 5. Following Clare[14] the temperatures are normalised using

$$\theta = L(T - T')/\Delta T \tag{23}$$

where T is the measured temperature, T' is the temperature which would have been measured at the axial level of the blockage had the experiment been performed in the absence of a blockage, and ΔT is the mean channel temperature rise over the heated section (length L) if all the pins were heated. θ represents the length downstream of the blockage where a measured temperature would be obtained in the absence of a blockage in a fully rated cluster. For a particular geometry, θ is a measure of the local temperature rise due to the blockage and is independent of power, flow or blockage position.

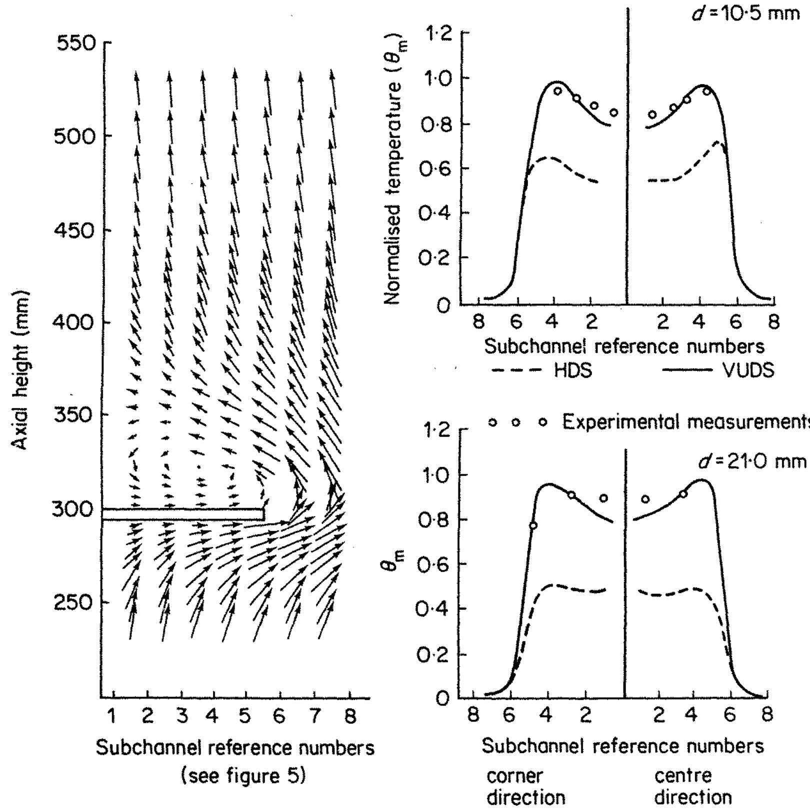

Figure 6. Calculational results

An important parameter in determining the temperature distribution in the wake is the turbulent exchange coefficient for heat. The results here have been obtained using a simple correlation based on a mixing length hypothesis of the form

$$\Gamma = \rho u' l \tag{24}$$

in which it is assumed that the velocity fluctuation u' is a given fraction α of the bulk flow u_s and the mixing length l a given fraction β of the subchannel diameter D. The fluctuations of the velocity about the mean value are typically small, e.g. $\alpha \sim 0{\cdot}1$ near the axis in pipe flow.[15] In addition, it would be expected that in subchannel geometry β would take values up to a maximum of unity. The numerical results were therefore obtained using a correlation of the form

$$\Gamma = f \rho u_s D \tag{25}$$

where $f(=\alpha\beta)$ is taken as an empirical constant.

In Figure 6 the continuous curve was obtained with the general form of the VUDS using values of $f = 0{\cdot}073$. The broken curve was obtained with the HDS using a similar value. But the same curve, on the resolution illustrated, is indistinguishable from a corresponding curve obtained with the HDS using an alternative prescription for Γ based on a pipe flow correlation,[10] which typically gives values $\frac{1}{60}\Gamma$ in the Reynolds number range of interest. The explanation is that the temperature rise in HDS is almost entirely a function of the false diffusion and essentially independent of the impressed real diffusion. In contrast, the results for the VUDS exhibit the expected dependency on Γ and the curve illustrated approximately spans the experimental results.

DISCUSSION

It has been conjectured that in certain circumstances the most general form of the VUDS described here may produce oscillations in the solution at sufficiently high Peclet number. The proposed explanation is the dependency of the general scheme, in contrast to the simpler version, on nodes downstream of the direction of differencing in the case when the flow angle lies outside the range $\frac{1}{2} \leq \left|\frac{v}{u}\right| \leq 2$ (cf. unidirectional flow problem with equal mesh described earlier). However, calculations for a wide range of flows have been performed at Winfrith using the SABRE code, incorporating the full VUDS treatment of the energy equation, and there has been little tendency towards oscillation in the solution or difficulties in procuring convergence. Under few circumstances therefore, in the treatment of the energy equation, has it been necessary to resort to the simpler and slightly less accurate version.

Nevertheless recent work aimed at incorporating a VUDS treatment of the flow equations has indicated that convergence of the general scheme has been rather poor in certain elliptic flow regimes and in these circumstances it has been necessary to resort to the simpler version. The reasons are at present unclear, but it is possible that the performance of the VUDS in linear systems may be different than in nonlinear systems in respect of these phenomena (i.e. convergence behaviour, tendency to produce numerical oscillation). This is a subject of further research.

CONCLUSIONS

Unlike the UDS, the accuracy of the most general form of the VUDS described gives an approximation of the convective transport terms at high Peclet number which is not impaired by false diffusion when the flow is oblique to mesh. The simplified form (which corresponds to one given by Raithby), together with the source correction term, is slightly less accurate but is a marked improvement on the UDS. This is demonstrated by comparison of the finite difference schemes' predictions with an analytic solution on a model vortex problem specifically chosen to highlight any errors in the schemes which arise as a consequence of false diffusion. By reverting to the simplified form if necessary, the solutions can be guaranteed oscillation free in all circumstances and therefore in this respect the VUDS is an improvement on the CDS. In addition, iterative solution techniques are available by application of the deferred corrector technique.

The half-mesh displacement inadequacies inherent in the VUDS may be eliminated using the source corrector technique. The VUDS has been tested successfully on a large-scale industrial application in which the more commonly used methods employing UDS or CDS approximations are for various reasons inaccurate or unsuitable.

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